

I.D. #	
Name	Philip Phillips
Subject	
Course	Phys. 560
Section	
Instructor	
Date	

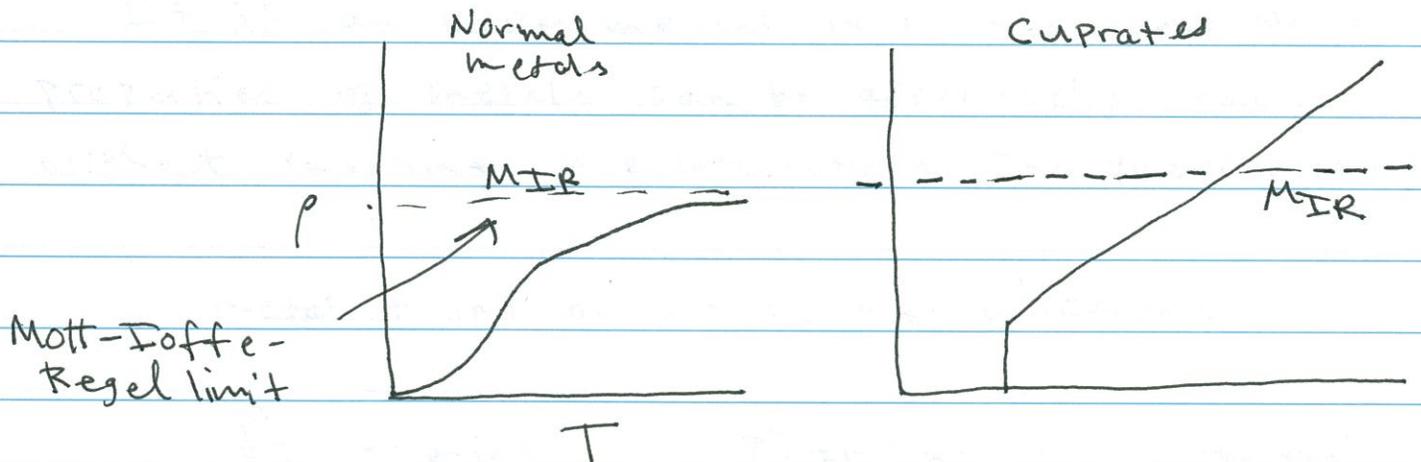
Receiving or giving aid in a final examination is a cause for dismissal from the University.

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Lecture 16:

A.) Review

e^- + Phonons give rise to a resistivity that scales as T^5 for $T \ll T_D$ and T for $T \gg T_D$. Once the mean-free path drops to the interatomic spacing, the resistivity must saturate.



As I said the origin of T -linear resistivity is a mystery. Here's a little observation.

What is the most fundamental length in the universe? just combine, c , G and \hbar

$$l_p = \sqrt{\hbar G / c^3}$$

=> The Planck time is \hbar/c . What if we were asked to come up with a thermal equivalent of this quantity. We would have to introduce k_B .

$$\boxed{\frac{\hbar}{\tau} = k_B T} \sim \rho$$

This is the Planckian dissipation rule. There is no scale on the R.H.S. Is this what T-linear resistivity is about? Let's see what happens in a F.L.

B.) Fermi Liquid Theory:

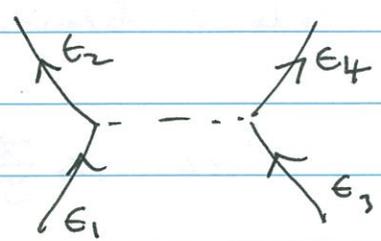
It is an experimental fact that most electronic properties of metals can be accurately known without invoking e-e-interactions. The question is why?

metal + interactions = free electrons.

$$\frac{p^2}{2m} + e-int = p^2/2m^* \leftarrow \text{This is the real statement.}$$

A key aspect of the interactions is that they are short-range. Why do short-range interactions not matter? Recall a F.L. is in momentum space.
=> It is local in momentum space. => It is

non-local in real space. \Rightarrow to destroy a FL one needs
 to add something in momentum space that is local.
 \Rightarrow it must be non-local in real space. This in a
 nutshell such short-range interactions do nothing to
 destroy a F.L. Let's look at this further. Here is
 a typical diagram for e-e scattering



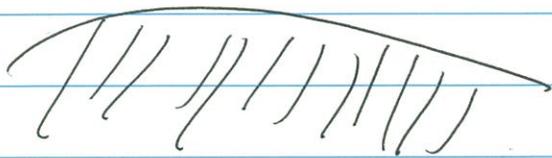
Energy conservation implies that $E_{initial} = E_{final}$.

$$E_{int} = E_1 + E_3$$

$$E_{final} = E_2 + E_4$$

$$\Rightarrow E_1 + E_3 = E_2 + E_4$$

So how many independent degrees of freedom are there? Energy is well defined only if a particle picture is valid. This is only valid for free field theory. So consider a collection of electrons on a Fermi surface. Recall $E_{tot} = \sum_i E_i = \sum_{\vec{p}} \frac{p^2}{2m}$.



Once we pick ϵ_1 , there are only 2 degrees of freedom that can be chosen freely. Let's call these ϵ_2 and $\epsilon_4 \Rightarrow \epsilon_3$ is now fixed. Let's say that the width of states from which ϵ_1 is chosen is $\epsilon_1 - \epsilon_F = \Delta\epsilon$. \Rightarrow the number of choices for ϵ_2 and ϵ_4 should scale as $(\Delta\epsilon)(\Delta\epsilon)$. \Rightarrow the scattering rate should scale as $\Delta\epsilon^2$

$$\Rightarrow \frac{1}{\tau} \propto \Delta\epsilon^2$$

but $\Delta\epsilon \sim k_B T$.

$$\Rightarrow \frac{1}{\tau} \propto (k_B T)^2$$

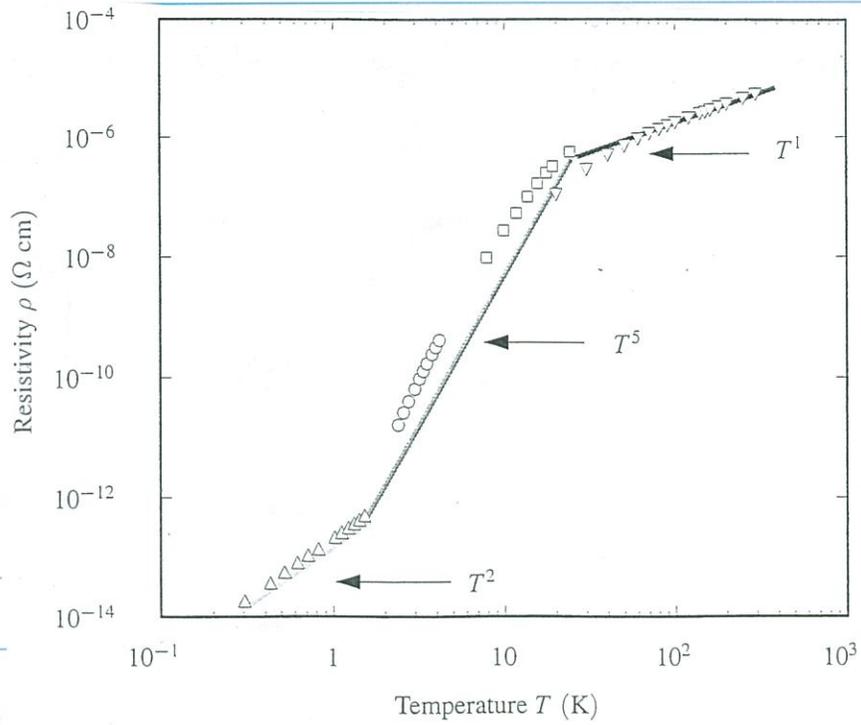
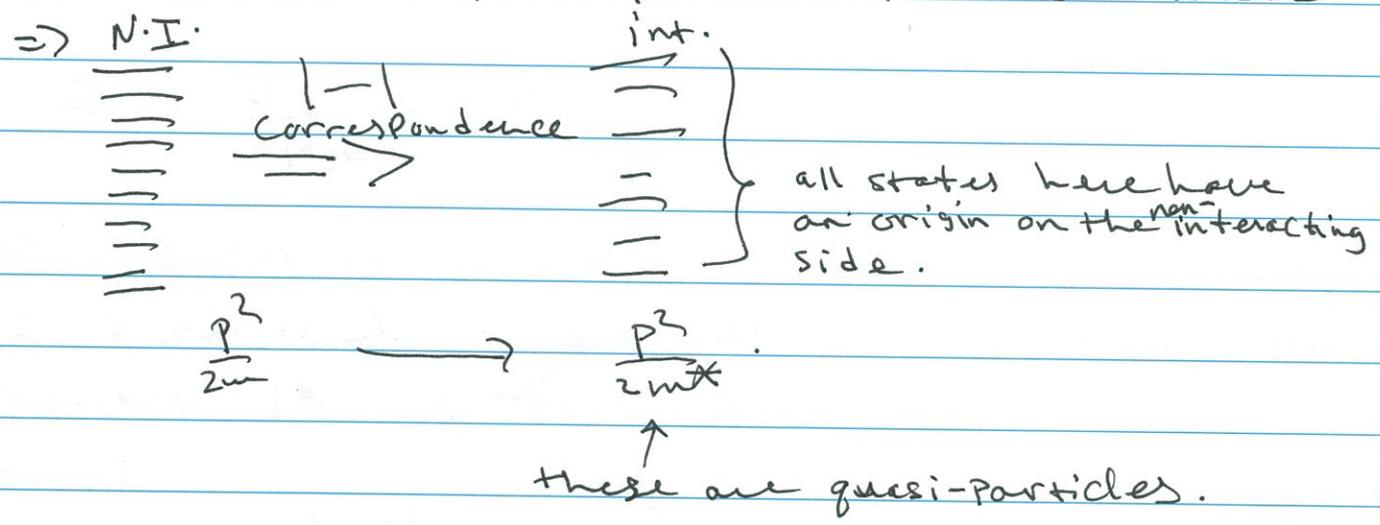


Figure 18.1. Resistivity of potassium from 0.1 to 300 K. At the lowest temperatures, the resistance varies as T^2 , from 2 to 20 K, it varies as T^5 , and from 100 K upwards, it varies as T^{-1} .

Energy scale enters the Planckian dissipation formula because of the linear in T dependence. Hence the T^2 requires a $1/\epsilon$ dependence.

In general the P-states do scatter \Rightarrow they have a certain width. $\tilde{\Sigma}_p = \epsilon_p + i\hbar/\tau$. The Landau condition for the state with momentum P to be an eigenstate is that $\frac{\hbar}{\tau} \ll \tilde{\Sigma}_p \Rightarrow$ although electron \rightarrow interact, the particle picture still holds.



Here's a way of thinking about F.L.T. At room T, $(k_B T)^2 \sim 10^{-4} \text{ eV} \Rightarrow \tau \sim 10^{10} \text{ s}$. A typical relaxation ϵ_F

time in a metal is much shorter. \Rightarrow relaxation in metals really has nothing to do with e-e-int.

We can formalize this by doing renormalization group theory. The key here is to integrate out the high-energy states to obtain a true low-energy theory. The end is a recursion relation for how the

coupling constant evolves as the energy decreases \Rightarrow .
The result is

$$\frac{dg}{d \ln E} = g^2 \equiv \beta(g)$$

\uparrow this is called the β -fcn.

$d \ln E$ is negative because we are decreasing the energy scale.

$$\Rightarrow \frac{dg}{d \ln E} = -g^2$$

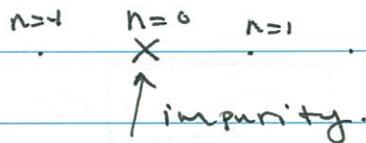
\Rightarrow if g is initially positive \Rightarrow as the energy is decreased g decreases and eventually vanishes.
However if $g \leftarrow 0 \Rightarrow$

$$\frac{d|g|}{d \ln E} = +g^2 \Rightarrow g \text{ grows.}$$

This caused an instability, the one discovered by Cooper that led to the BCS theory. BCS is the answer to superconductivity because of this equation. If an attraction exists between the electrons, the ground state becomes unstable \Rightarrow new state of matter.

C.) Impurities

Before we introduce SC, let's treat the problem of impurities in a metal.



The Schrodinger equation is

$$E c_n = -t(c_{n+1} + c_{n-1}) + \epsilon_n c_n.$$

Let $\epsilon_n = \begin{cases} 0 & |n| > 0 \\ W & n = 0. \end{cases}$

The wave function looks like

$$c_n = \begin{cases} e^{ikn} + R e^{-ikn} & n < 0 \\ T e^{ikn} & n > 0 \end{cases}$$

The unknowns are C_0 , R , and T .

Site -1:

$$E c_{-1} = -t(c_0 + c_{-2}).$$

$$\uparrow E = -2t \cos k$$

$$\begin{aligned}
 (e^{ik} + e^{-ik})(e^{-ik} + R e^{ik}) &= C_0 + e^{-2ik} + R e^{2ik} \\
 1 + e^{-2ik} + R + R e^{2ik} &= C_0 + e^{-2ik} + R e^{2ik} \\
 \Rightarrow \boxed{C_0 = 1 + R}
 \end{aligned}$$

Site 1: $EC_1 = -t(C_0 + C_2)$

$$(e^{ik} + e^{-ik})Te^{ik} = C_0 + Te^{2ik}$$

$$\Rightarrow \boxed{C_0 = T} \quad \text{we need more info.}$$

$$\Rightarrow T = 1 + R$$

Site 0: $(E - W)C_0 = -t(C_1 + C_{-1})$

$$(e^{ik} + e^{-ik} + \frac{W}{t})(1 + R) = e^{ik}(1 + R) + (e^{-ik} + Re^{ik})$$

$$Re^{-ik} + \frac{W}{t}(1 + R) = Re^{ik}$$

$$\Rightarrow (-2i \sin k + \frac{W}{t})R = -W/t$$

$$\boxed{R = \frac{-W/t}{\frac{W}{t} - 2i \sin k}}$$

$$\Rightarrow |R|^2 = \frac{(W/t)^2}{(W/t)^2 + 4 \sin^2 k}$$

\Rightarrow regardless of the strength of the impurity, there is always a scattered state. \Rightarrow the e^- states scatter destructively.